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Semiclassical Model of Quark–Gluon Plasma

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A semiclassical model of a gluon–quark plasma (QGP) is provided. The spin of the quark is described by an antisymmetric tensor $S^{\mu\nu}$ and color charges, and spin tensor are defined as semiclassical numbers. The transport properties of QGP in color space and spin space are investigated. The consistency of the semiclassical model and the quantum models of QGP is discussed.

1. INTRODUCTION

It is predicted by lattice gauge theory [1, 2] and finite-temperature field theory [3] that the deconfinement phase transition of the hadron can occur at high temperature and density and a quark–gluon plasma (QGP) can be produced. The critical temperature of the deconfinement phase transition is about 200 Mev [4, 5], which is so high that the QGP is expected to be found in relativistic heavy-ion collisions. After production in a relativistic heavy-ion collision, the QGP will go toward to equilibrium. The transport process of the QGP is a very important physical process and has been studied by many physicists [e.g., 6, 7] using classical and quantum models of QGP. Generally, the classical model is consistent with the quantum model. Heinz and coworkers have discussed consistency when the spin of the quark is not considered. In this paper, we discuss consistency when the spin of the quark is retained. In order to realize our aim, a semiclassical model of QGP is presented in Section 2. In Section 3 we expand the semiclassical limit equations of the quantum model of QGP in color and spin spaces. In Section 4, we discuss the consistency of the two models.

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2. SEMICLASSICAL MODEL OF QUARK–GLUON PLASMA

As a gluon is a non-Abelian particle, the non-Abelian property is important in the QGP and makes the color Q^a noncommutative. So we assume the color charges do not commute with each other in the semiclassical model of QGP. This differs from the classical model where the color charges are commutative. Except for the noncommutation of the color charges, we still define the color charges as satisfying the Lee algebra of SU(3) in color space.

In the QGP, when the spin of the quark is considered, there is an interaction of the quark spin and the gluon field, which is given by

$$\mathcal{H}_I = \frac{g}{2}(D_\nu S^{\alpha\beta} F_{\alpha\beta})^a Q^a \tag{1}$$

The interaction makes the properties of the QGP in color and spin spaces very complex. In the semiclassical model of the QGP, we use $S^{\mu\nu}$ (antisymmetric tensor) to describe the spin of the quark and also assume that the components of the $S^{\mu\nu}$ are not commutative. In ref. 8, the motion equation of $S^{\mu\nu}$ is given by

$$m\frac{dS^{\mu\nu}}{d\tau} = gQ_a[F^{\mu}_{a\lambda}S^{\lambda\nu} - F^{\nu}_{a\lambda}S^{\lambda\mu}] \tag{2}$$

In the semiclassical model of the QGP, a quark is described by one-particle distribution function, which is a function of dynamic variables, $f = f(x, p, Q, S)$, and denotes the probability density of finding a quark at given time-space point (x, p, Q, S) in phase space. In ref. 9, $\dot{p}^\mu$ $\dot{Q}^a$ are given by

$$m\dot{p}^\nu = m\frac{\partial p^\mu}{\partial\tau} = -gQ^a p^\mu F^a_{\mu\nu} + \frac{g}{2}(D_\nu S^{\alpha\beta}F_{\alpha\beta})^a Q^a \tag{3}$$

$$m\dot{Q}^a = m\frac{\partial p^\mu}{\partial\tau} = -qg_{abc}\left(p^\mu A^b_\mu + \frac{1}{2}S^{\alpha\beta}F^b_{\alpha\beta}\right)Q_c \tag{4}$$

The evolution of the one-particle distribution function with proper time is given by

$$\begin{aligned} p^\mu\partial_\mu f(x, p, Q, S) = {} & [gQ^a p^\mu F^a_{\mu\nu} - \frac{g}{2}(D_\nu S^{\alpha\beta}F_{\alpha\beta})^a Q^a]\,\partial^\nu_p f(x, p, Q, S) \\ & + \left[gf_{abc}\left(p^\mu A^b_\mu + \frac{1}{2}S^{\alpha\beta}F^b_{\alpha\beta}\right)Q_c\right]\partial^a_Q f(x, p, Q, S) \\ & - [gQ_a(F^a_{\mu\lambda}S^\lambda_\nu - F^a_{\nu\lambda}S^\lambda_\mu)]\,\partial^{\mu\nu}_S f(x, p, Q, S) \\ & + C(x, p, Q, S) \end{aligned} \tag{5}$$

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where $C(x, p, Q, S)$is a collision term describing short-range two-body collisions. Here $\partial_p^\nu = \partial/\partial p_\nu$, $\partial_Q^a = \partial/\partial Q^a$, and $\partial_S^{\mu\nu} = \partial/\partial S_{\mu\nu}$. Equation (5) is called the classical transport equation of the quark.

Because $f(x, p, Q, S)$ is related to Q^a and the color charges are noncommutative with each other, we assume that the distribution function does not commute with color charges, so that the product of f and Q^a is related to order of f and Q^a. For same reason, the product of f and $S^{\mu\nu}$ also is related to the order of f and $S^{\mu\nu}$. In Eq. (5), we write Q^a and $S^{\mu\nu}$ on the left of f and call it the left product form of the classical transport equation. The transport equation still can been written in the right product form, or Q^a and $S^{\mu\nu}$ placed on the right side of f,

$$\begin{aligned} p^\mu \partial_\mu f(x, p, Q, S) = {} & \partial_p^\nu f(x, p, Q, S)\left[g Q^a p^\mu F^a_{\mu\nu} - \frac{g}{2}(D_\nu S^{\alpha\beta} F_{\alpha\beta})^a Q^a \right] \\ & + \partial_Q^a f(x, p, Q, S)\left[g f_{abc}\left(p^\mu A^b_\mu + \frac{1}{2} S^{\alpha\beta} F^b_{\alpha\beta} \right) Q_c \right] \\ & - \partial_S^{\mu\nu} f(x, p, Q, S)[g Q_a (F^a_{\mu\lambda} S^\lambda_\nu - F^a_{\nu\lambda} S^\lambda_\mu)] \\ & + C(x, p, Q, S) \end{aligned} \tag{6}$$

We can write the transport equation in a symmetric form,

$$\begin{aligned} p^\mu\, \partial_\mu f = {} & \frac{g}{2} p^\mu F^a_{\mu\nu}\{Q^a, \partial_p^\nu f\} - \frac{g}{4}(D_\nu F_{\alpha\beta})^a \{S^{\alpha\beta} Q^a, \partial_p^\nu f\} \\ & + \frac{g}{2} f_{abc} p^\mu A^b_\mu \{Q^c, \partial_Q^a f\} + \frac{g}{4} f_{abc} F^b_{\alpha\beta}\{S^{\alpha\beta} Q_c, \partial_Q^a f\} \\ & - \frac{g}{2}\{Q_a (F^a_{\mu\lambda} S^\lambda_\nu - F^a_{\nu\lambda} S^\lambda_\mu), \partial_S^{\mu\nu} f\} + C(x, p, Q, S) \end{aligned} \tag{7}$$

In color space, we can define the color moments by the distribution function

$$f_\nu(x, p, S) = \int b f(x, p, Q, S)\, dQ \tag{8}$$

In the above equation, the integral in color space needs to be defined. Considering that the color charge Q^a should be orthonormal in color space, we define the orthonormal relations

$$\int Q^a\, dQ = 0, \qquad \int Q^a\, Q^b\, dQ = \delta_{ab} \tag{9}$$

The distribution function is expanded in color space as

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$$f = f_0\, I + f_a Q^a \tag{10}$$

In the classical model, the expansion above is not complete and Q^aQ^b, $Q^aQ^bQ^c$, . . . need to be considered. Here we do not need to consider them because these terms can be expanded in the form of Eq. (10). For example,

$$Q^aQ^b = \frac{1}{6}\,\delta_{ab} - \frac{1}{2}\,(d_{abc} + if_{abc})Q^c \tag{11}$$

Using Eq. (9), have $f_0 = \int f\, dQ$, $f_a = \int Q^a\, f\, dQ$, $= \int fQ^a\, dQ$. Here f_0 and f_a are called the color singlet and color octet distribution functions, respectively. Using Eq. (11), we find the relations

$$\int Q^aQ^bf\, dQ = \int Q^bfQ^a\, dQ = \int fQ^aQ^b\, dQ = \frac{f_0}{6}\,\delta_{ab} - \frac{1}{2}\,(d_{abc} + if_{abc})\, f_c \tag{12}$$

Using the transport equations, we can construct the color moment equations. Using Eq. (5), we obtain the left product form of the color moment equations,

$$\begin{aligned} p^\mu\, \partial_\mu f_0\,(x, p, S) &= \left[gp^\mu F^a_{\mu\nu} - \frac{g}{2}\,(D_\nu S^{\alpha\beta}F_{\alpha\beta})^a\right]\partial^\nu_p f_a(x, p, S) \\ &\quad - [g(F^a_{\mu\lambda}S^\lambda_\nu - F^a_{\nu\lambda}S^\lambda_\mu)]\partial^{\mu\nu}_S f_a(x, p, S) + C(x, p, S) \end{aligned} \tag{13}$$

$$\begin{aligned} p^\mu\, \partial_\mu f_a(x, p, S) &= \left[gp^\mu F^b_{\mu\nu} - \frac{g}{2}\,(D_\nu S^{\alpha\beta}F_{\alpha\beta})^b\right]\partial^\nu_p f_{ab}(x, p, S) \\ &\quad - \left[gf_{abc}\left(p^\mu A^b_\mu + \frac{1}{2}\,S^{\alpha\beta}F^b_{\alpha\beta}\right)\,\partial f_c(x, p, S)\right. \\ &\quad - [g(F^b_{\mu\lambda}S^\lambda_\nu - F^b_{\nu\lambda}S^\lambda_\mu)]\,\partial^{\mu\nu}_S f_{ab}(x, p, S) \\ &\quad + C_a(x, p, S) \end{aligned} \tag{14}$$

Using Eq. (6), we obtain the right form of the color moment equations

$$\begin{aligned} p^\mu\, \partial_\mu f_0(x, p, S) &= \partial^\nu_p f_a(x, p, S)\left[gp^\mu F^a_{\mu\nu} - \frac{g}{2}\,(D_\nu S^{\alpha\beta}F_{\alpha\beta})^a\right] \\ &\quad - f_a(x, p, S)[g(F^a_{\mu\lambda}S^\lambda_\nu - F^a_{\nu\lambda}S^\lambda_\mu)]\,\partial^{\mu\nu}_S + C(x, p, S) \end{aligned} \tag{15}$$

$$p^\mu\, \partial_\mu f_a(x, p, S) = \partial^\nu_p f_{ab}(x, p, S)\left[gp^\mu F^b_{\mu\nu} - \frac{g}{2}\,(D_\nu S^{\alpha\beta}F_{\alpha\beta})^b\right]$$

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$$- \partial f_c(x, p, S)\left[g f_{abc}\left(p^\mu A^b_\mu + \frac{1}{2} S^{\alpha\beta} F^b_{\alpha\beta} \right)\right]$$
$$- \partial^{\mu\nu}_S f_{ab}(x, p, S)[g(F^b_{\mu\lambda} S^\lambda_\nu - F^b_{\mu\lambda} S^\lambda_\mu)]$$
$$+ C_a(x, p, S) \tag{16}$$

These color moment equations reflect the transport properties of the QGP in color space. In the classical model of the QGP, these color moment equations form a BBGKY hierarchy of coupled equations because in this model the color charges commute with each other. The hierarchy of color moment equations can be truncated by hand by imposing some conditions. In the semiclassical model, the hierarchy of color moment equations is truncated automatically and cannot form a hierarchy of coupled equations.

The properties of the distribution function in spin space will be discussed in Section 4.

3. TRANSPORT PROPERTIES OF THE QUARK IN COLOR AND SPIN SPACE

In the quantum model of the QGP, a quark is described by a Wigner function and its quantum transport equation [9]

$$p^\mu D_\mu \hat{W}(x, p)$$
$$= -\frac{g}{2} p^\mu \partial^\nu_p \int_0^1 ds\, [(e^{-s\Delta} F_{\mu\nu})\hat{W}(x, p) + \hat{W}(x, p)(e^{s\Delta} F_{\mu\nu})]$$
$$+ \frac{ig}{4} [\sigma^{\mu\nu}(e^{-\Delta} F_{\mu\nu})\hat{W}(x, p) - \hat{W}(x, p)(e^{\Delta} F_{\mu\nu})\sigma^{\mu\nu}]$$
$$+ \frac{ig}{4} \partial^\nu_p \int_0^1 ds\, s[(e^{-s\Delta} F_{\mu\nu}) D^\mu \hat{W}(x, p) - D^\mu \hat{W}(x, p)(e^{s\Delta} F_{\mu\nu})]$$
$$- \frac{i}{8} g^2 \partial^\mu_p \partial^\nu_p \int_0^1 ds\, s \int_0^1 d\bar{s}\, \{(e^{-s\Delta} F_{\mu\eta})[(e^{-\bar{s}\Delta} F^\eta_\nu)\hat{W}(x, p)$$
$$+ \hat{W}(x, p)(e^{\bar{s}\Delta} F^\eta_\nu)] - [(e^{-\bar{s}\Delta} F^\eta_\nu)\hat{W}(x, p)$$
$$+ \hat{W}(x, p)(e^{\bar{s}\Delta} F^\eta_\nu)](e^{s\Delta} F_{\mu\eta})\} \tag{17}$$

where the triangle operator is defined as $\Delta = (i/2)\, \partial^\nu_p D_\nu(x)$ and $S^{\mu\nu} = \frac{1}{2}\sigma^{\mu\nu}$. The following products of operators are defined:

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$$\hat{O}A \otimes B = A \otimes \hat{O}B, \qquad A \otimes B\hat{O} = A\hat{O} \otimes B \tag{18}$$

The quantum transport equation of the quark is very complex and can yield much information. First we investigate the properties of the QGP in color and spin spaces. For this we need the semiclassical limit equation of Eq. (17), which is given by [6, 9]

$$p^{\mu} D_{\mu} W(x, p) + \frac{g}{2} p^{\mu} \partial_p^{\nu}\{F_{\mu\nu}, W(x, p)\} - \frac{ig}{4} [\sigma^{\mu\nu} F_{\mu\nu}, W(x, p)]$$
$$- \frac{g}{8} \{D_{\nu}\sigma^{\alpha\beta}F_{\alpha\beta}, \partial_p^{\nu} W(x, p)\} = 0 \tag{19}$$

where we only retain the first-order derivative terms of the Wigner function for the distribution function, which are supposed to vary slowly in nonequilibrium statistical mechanics [10], and the collision terms are not written out.

In Eq. (19), the color charges are 3×3 matrices ($Q^a = -\lambda^a/2$) in color space, and the spin tensors $S^{\mu\nu}$ are 4×4 matrices in spin space. The Wigner function $W(x, p)$ is a matrix in the space of the direct product of the color and spin, which is a 3×3 matrix in the color space and a 4×4 matrix in the spin space. $W(x, p)$ is a function of x and p, and the color properties and the spin properties are determined by the matrix forms of $W(x, p)$ in color space and in spin space.

In the color space, $W(x, p)$ is a matrix of the fundamental representation of the SU(3) group and can be reduced to the direct sum of a one-dimension irreducible representation and an eight-dimenstion irreducible representation [11],

$$W(x, p) = W_0 \frac{I}{3} - W^{\mu}\lambda_a \tag{20}$$

where $W_0 = T_r(W(x, p))$ and $W_a = T_r(W(x, p)Q^a)$ correspond to color singlet and color octets of $W(x, p)$; T_r denotes taking the trace in color space. Taking the trace of Eq. (19), obtain

$$p^{\mu} \partial_{\mu} W_0 = \frac{g}{c} p^{\mu} F^a_{\mu\nu} \partial_p^{\nu} W_a - \frac{ig}{4} [\sigma^{\alpha\beta} F^a_{\alpha\beta}, W_a]$$
$$- \frac{g}{8} \{(D_{\nu}F_{\alpha\beta})^a \sigma^{\alpha\beta}, \partial_p^{\nu} W_a \tag{21}$$

where

$$(D_{\nu}F_{\alpha\beta})^a = \partial_{\nu} F^a_{\alpha\beta} + f^{abc} A^b_{\nu} F^c_{\alpha\beta} \tag{22}$$

Timing Eq. (19) by Q^a, then taking the trace in color space, have

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$$p^{\mu}\,\partial^{\mu}W_a = gp^{\mu}F^{b}_{\mu\nu}\,\partial^{\nu}_{p}W_{ab} - gf_{abc}p^{\mu}A^{b}_{\mu}W_c - \frac{ig}{4}\,[\sigma^{\alpha\beta}F^{b}_{\alpha\beta}T_r(WQ^aQ^b) - T_r(WQ^bQ^a)F^{b}_{\alpha\beta}\sigma^{\alpha\beta}] - \frac{g}{8}\,(D_{\nu}F_{\alpha\beta})^b[\sigma^{\alpha\beta}\,\partial^{\nu}_{p}T_r(WQ^aQ^b) + \partial^{\nu}_{p}\,T_r(WQ^bQ^a)\sigma^{\alpha\beta]} \qquad (23)$$

where

$$W_{ab} = T_r(W\{Q^a, Q^b\}/2) = \frac{\delta_{ab}}{6}\,W_0 - \frac{1}{2}\,d_{abc}W_c \qquad (24)$$

d_{abc} is a symmetry construct of the SU(3) group.

In Eqs. (21) and (23), W_0, W_a, and $Tr(WQ_aQ_b)$ are matrices in spin space. They are not commutative with $\sigma^{\mu\nu}$. In the spin space, there are 16 independent γ matrices [6],

$$1, \quad \gamma^{\mu}, \quad i\gamma^5, \quad \gamma^{\nu}\gamma^5, \quad \sigma^{\mu\nu}$$

W_0, W_a, . . . can be expanded in the 16 γ matrices,

$$W_i = \frac{1}{4}\,(B_i + i\gamma^5P_i + \gamma^{\mu}V^{i}_{\mu} + \gamma^{\mu}\gamma^5D^{i}_{\mu} + \sigma^{\mu\nu}T^{i}_{\mu\nu}) \qquad (25)$$

where $B_i = t_r(W_i)$, $P_i = t_r(-i\gamma^5W_i)$, $V^{i}_{\mu} = t_r(\gamma_{\mu}W_i)$, $D^{i}_{\mu} = t_r(\gamma^5\gamma_{\mu}W_i) = t_r(W_i\gamma^5\gamma_{\mu})$, $T^{i}_{\mu\nu} = t_r(W_iS_{\mu\nu})$ $(i = 0, a)$ are scalar, vector, pseudoscalar, pseudovector, and tensor, respectively, and t_r denotes taking the trace in spin space. Decomposing Eq. (21) in spin space, we obtain a set of color singlet equations

$$p^{\mu}\,\partial_{\mu}B_0 = gp^{\mu}F^{a}_{\mu\nu}\,\partial^{\nu}_{p}B_a - \frac{g}{2}\,(D_{\nu}F_{\alpha\beta})^a\,\partial^{\nu}_{p}T^{\alpha\beta}_{a} \qquad (26)$$

$$p^{\mu}\,\partial_{\mu}P_0 = gp^{\mu}F_{\mu\nu}\,\partial^{\nu}_{p}P_a - \frac{g}{2}\,(D_{\nu}\bar{F}^{a}_{\alpha\beta}\,\partial^{\nu}_{p}T^{\alpha\beta}_{a}) \qquad (27)$$

where $\bar{F}_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\mu\nu}F^{\mu\nu}$,

$$p^{\mu}\,\partial_{\mu}V^{\lambda}_{0} = gp^{\mu}F^{a}_{\mu\nu}\,\partial^{\nu}_{p}V^{\lambda}_{a} - gV^{\rho}_{a}F^{a\lambda}_{\rho} - \frac{g}{2}\,(D_{\nu}\bar{F}^{\lambda\rho})^a\,\partial^{\nu}_{p}D^{a}_{\rho} \qquad (28)$$

$$p^{\mu}\,\partial_{\mu}D^{\lambda}_{0} = gp^{\mu}F^{a}_{\mu\nu}\,\partial^{\nu}_{p}D^{\lambda}_{a} - gD^{\rho}_{a}F^{a\lambda}_{\rho} - \frac{g}{2}\,(D_{\nu}\bar{F}^{\lambda\rho})^a\,\partial^{\nu}_{p}V^{a}_{\rho} \qquad (29)$$

$$p^{\mu}\,\partial_{\mu}T^{\lambda\rho}_{0} = gp^{\mu}F^{a}_{\mu\nu}\,\partial^{\nu}_{p}T^{\lambda\rho}_{a} - \frac{g}{2}\,(D_{\nu}F_{\alpha\beta})^a\,\partial^{\nu}_{p}W^{\alpha\beta,\lambda\rho}_{a} + g(F^{\lambda\sigma}_{a}T^{a\rho}_{\sigma} - F^{\rho\sigma}_{a}T^{a\lambda}_{\sigma}) \qquad (30)$$

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where

$$W_a^{\alpha\beta,\lambda\rho} = t_r(W_a\{S^{\alpha\beta}, S^{\lambda\rho}\}/2) = -\frac{1}{4} P_a \epsilon^{\alpha\beta\lambda\rho} - \frac{1}{4} B_a(g^{\alpha\rho}g^{\beta\lambda} - g^{\alpha\lambda}g^{\beta\rho}) \tag{31}$$

Decomposing Eq. (23), we obtain another a set of color octet equations,

$$p^\mu \partial_\mu B_a = gp^\mu F^b_{\mu\nu} \partial^\nu_p B_{ab} - \frac{g}{2}(D_\nu F_{\alpha\beta})^b \partial^\nu_p T^{\alpha\beta}_{ab} - gf_{abc}p^\mu A^b_\mu B_c - \frac{g}{2} f_{abc}F^b_{\mu\nu}T^{\mu\nu}_c \tag{32}$$

where

$$B_{ab} = T_r t_r(W\{Q^a, Q^b\}/2) = \frac{\delta_{ab}}{6} B_0 - \frac{1}{2} d_{abc}B_c \tag{33}$$

$$T^{\alpha\beta}_{ab} = T_r t_r(W\{Q^a, Q^b\}S^{\alpha\beta}/2) = \frac{\delta_{ab}}{6} T^{\alpha\beta}_0 - \frac{1}{2} d_{abc}T^{\alpha\beta}_c \tag{34}$$

$$p^\mu \partial_\mu P_a = gp^\mu F^b_{\mu\nu} \partial^\nu_p P_{ab} - gf_{abc}p^\mu A^b_\mu P_c - \frac{g}{2} f_{abc}\bar{F}^b_{\alpha\beta}T^{\alpha\beta}_c - \frac{g}{2}(D_\nu \bar{F}_{\alpha\beta})^b T^{\alpha\beta}_{ab} \tag{35}$$

$$p^\mu \partial_\mu V^\lambda_a = gp^\mu F^b_{\mu\nu} \partial^\nu_p V^\lambda_{ab} - gf_{abc}p^\mu A^b_\mu V^\lambda_c + gF^{\lambda}_{b\rho}V^\rho_{ab} + \frac{g}{2} f_{abc}(\bar{F}^{\rho\lambda}_b)D^c_\rho + \frac{g}{2}(D_\nu \bar{F}^\lambda_\rho)^b \partial^\nu_p D^\rho_{ab} - \frac{g}{4} f_{abc}(D_\nu F^{\lambda\rho})^b \partial^\nu_p V^c_\rho \tag{36}$$

$$p^\mu \partial_\mu D^\lambda_a = gp^\mu F^b_{\mu\nu} \partial^\nu_p D^\lambda_{ab} - gf_{abc}p^\mu A^b_\mu D^\lambda_c + \frac{g}{2} f_{abc}(\bar{F}^{\rho\lambda}_b)V^c_\rho + gF^\lambda_{b\rho}D^\rho_{ab} + \frac{g}{2}(D_\nu \bar{F}^{\rho\lambda})^b \partial^\nu_p V^{ab}_\rho + \frac{g}{4} f_{abc}(D_\nu F^{\rho\lambda})^b \partial^\nu_p D^c_\rho \tag{37}$$

$$p^\mu \partial_\mu T^{\lambda\rho}_a = gp^\mu F^b_{\mu\nu} \partial^\nu_p T^{\lambda\rho}_{ab} - \frac{g}{2}(D_\nu F_{\alpha\beta})^b \partial^\nu_p W^{\alpha\beta,\lambda\rho}_{ab} - gf_{abc}p^\mu A^b_\mu T^{\lambda\rho}_c$$

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$$- \frac{g}{2} f_{abc} F^{b}_{\alpha\beta} W^{\alpha\beta,\lambda\rho}_{c} + g(F^{\lambda}_{b\nu} T^{\nu\rho}_{ab} - F^{\rho}_{b\nu} T^{\nu\lambda}_{ab})$$

$$- \frac{g}{4} f_{abc}[(D_\nu F^{\lambda}_{\sigma})^b \, \partial^{\nu}_{p} T^{\sigma\rho}_{c} - (D_\nu F^{\rho}_{\sigma})^b \, \partial^{\nu}_{p} T^{\sigma\lambda}_{c}] \tag{38}$$

where

$$W^{\alpha\beta,\lambda\rho}_{ab} = T_r t_r (W\{Q^a, Q^b\}\{S^{\alpha\beta}, S^{\lambda\rho}\}/4)$$
$$= -\frac{1}{4} \epsilon^{\alpha\beta\lambda\rho} \left(\frac{\delta_{ab}}{6} P_0 - \frac{1}{2} d_{abc} P_c \right)$$
$$- \frac{1}{4} (g^{\alpha\rho} g^{\beta\lambda} - g^{\alpha\lambda} g^{\beta\rho}) \left(\frac{\delta_{ab}}{6} B_0 - \frac{d_{abc}}{2} B_c \right) \tag{39}$$

The 10 equations obtained above correspond respectively to the transport equations of the color singlet spin scalar, the color singlet spin vector, and so on. In the equations of the color octet P_a, V^{λ}_{a}, D^{λ}_{a} one has P_{ab}, V^{λ}_{ab}, D^{λ}_{ab}, which have relations similar to Eq. (33). For the spin variable, one has $W^{\alpha\beta,\lambda\rho}_{i}$ ($i = a, ab$) in the equations of the spin tensor. Due to the constraint of Clifford algebra in spin space, they are related to P_i and B_i [Eqs. (31), (39)]. Inserting these relations into the relevant equations, we obtain two sets of independent equations, one set of spin vector and one of spin pseudovector equations.

The QED plasma is an Abelian plasma. The semiclassical limit equation for the quark has been expanded in spin space in ref. 12. Under the condition where the electromagnetic field is a constant field or varies slowly, there is a simple relation between the vector distribution function and the scalar distribution function,

$$V^{\alpha} = p^{\alpha} B \tag{40}$$

The QGP is a non-Abelian plasma. The case is complex. First, we consider the color singlet, Eqs. (26) and (28). When a gluon field is a constant field or varies slowly, we have

$$(D_\nu F_{\alpha\beta})^a = \partial_\nu F^{a}_{\alpha\beta} + f^{abc} A^{b}_{\nu} F^{c}_{\alpha\beta} = f^{abc} A^{b}_{\nu} F^{c}_{\alpha\beta} \tag{41}$$

Equations (26) and (28) become

$$p^{\mu} \, \partial_{\mu} B_0 = g p^{\mu} F^{a}_{\mu\nu} \, \partial^{\nu}_{p} B_a - \frac{g}{2} f^{abc} A^{b}_{\nu} F^{c}_{\alpha\beta} \, \partial^{\nu}_{p} T^{\alpha\beta}_{a} \tag{42}$$

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and

$$p^{\mu}\,\partial_{\mu}V_0^{\lambda} = gp^{\mu}F^{a}_{\mu\nu}\,\partial_p^{\nu}V_a^{\lambda} - gV_a^{\rho}F_{\rho}^{a\lambda} - \frac{g}{2}f^{abc}A_{\nu}^{b}\overline{F}_c^{\lambda\rho}\,\partial_p^{\nu}D_{\rho}^{a} \qquad (43)$$

There are no simple relations $V_0^{\lambda} = p^{\lambda}B_0$ and $V_a^{\lambda} = n^{\lambda}B_a$ in the above equations because of the last term, which comes from the non-Abelian property of the gluon. The scalar and vector equations of the color octet have the same results. This means that the QGP is more complex than the QED plasma owing to the non-Abelian property of the gluon.

4. CONSISTENCY BETWEEN SEMICLASSICAL MODEL AND QUANTUM MODEL

Section 2 gives a semiclassical model of the QGP with the aim of making the model consistent with the quantum model of the QGP. In Section 3 we obtained 10 equations from the semiclassical limit equation. If the semiclassical model is consistent with the quantum model, these equations should be obtained from the semiclassical model. To take traces of the Wigner function in color a spin spaces means to take the average in color and spin spaces. In the semiclassical model, to take the average in color and spin spaces is to do integrals in color space and spin spaces by the distribution function,

$$\langle b \rangle = \int bf\,dQ\,dS \qquad (44)$$

The integral in color space is defined in Section 2. Now we would like to define the integral in spin space. In the semiclassical model, there are 6 independent components of $S^{\mu\nu}$ describing the spin of the quark. In the last section, the properties of the Wigner function in spin space are expressed by 16 independent γ matrices [13]. In order to make the semiclassical model consistent with the quantum one, the spin space needs to be extended to 16 dimensions in the semiclassical model. Hence we introduce 16 orthonormal basic vectors in spin space, Γ^i ($\Gamma^0 = I$, $i = 1, 2, \ldots, 15$), which satisfy the Clifford algebra of γ matrices and have the properties

$$(\Gamma^i)^2 = 1 \qquad \Gamma^i\Gamma^j = -\Gamma^j\Gamma^i \qquad (i \neq j, \quad i, j \neq 1) \qquad (45)$$

The orthonormal relations are defined by

$$\int \Gamma^0\,dS = 1, \qquad \int \Gamma^i\,dS = 0, \qquad \int \Gamma^i\Gamma^j\,dS = \delta_{ij} \qquad (i, j \neq 1) \quad (46)$$

The distribution function is expanded in spin space by

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$$f(x, p, Q, S) = f^0(x, p, Q)I + \sum_{i=1}^{15} f^i(x, p, Q)\Gamma^i \tag{47}$$

Using the orthonormal relations (46), we have $f^0 = \int f\, dS$, $f^i = \int \Gamma^i f\, dS = \int f\Gamma^i\, dS$, and we can prove the following relations:

$$\int \Gamma^i\Gamma^j f\, dS = \int f\Gamma^i\Gamma^j\, dS = \int \Gamma^j f\Gamma^i\, dS = f^0\,\delta^{ij} + f^k \int \Gamma^i\Gamma^j\Gamma^k\, dS \tag{48}$$

As in color space, we define spin moments by

$$f^i(x, p, Q) = \int \Gamma^i f\, dS = \int f\Gamma^i\, dS \tag{49}$$

From Eqs. (13)–(16), we can find the spin moment equations:

$$p^\mu\,\partial_\mu f_0^0 = gp^\mu F^a_{\mu\nu}\,\partial_p^\nu f_a^0 - \frac{g}{2}(D_\nu F_{\nu\beta})^a\,\partial_p^\nu f_a^{\alpha\beta} \tag{50}$$

$$\begin{aligned} p^\mu\,\partial_\mu f_0^{\lambda\rho} &= gp^\mu F^a_{\mu\nu}\,\partial_p^\nu f_a^{\lambda\rho} - \frac{g}{2}(D_\nu F_{\alpha\beta})^a\,\partial_p^\nu f_a^{\lambda\rho,\alpha\beta} \\ &\quad + g(F^\lambda_{a\sigma} f_a^{\sigma\rho} - F^\rho_{a\sigma} f_a^{\sigma\lambda}) \end{aligned} \tag{51}$$

$$\begin{aligned} p^\mu\,\partial_\mu f_a^0 &= gp^\mu F^b_{\mu\nu}\,\partial_p^\nu f_{ab}^0 - \frac{g}{2}(D_\nu F_{\alpha\beta})^b\,\partial_p^\nu f_{ab}^{\alpha\beta} \\ &\quad - gf_{abc}p^\mu A^b_\mu f_c^0 - \frac{g}{2} f_{abc}F^b_{\alpha\beta} f_c^{\alpha\beta} \end{aligned} \tag{52}$$

$$\begin{aligned} p^\mu\,\partial_\mu f_a^{\lambda\rho} &= gp^\mu F^b_{\mu\nu}\,\partial_p^\nu f_{ab}^{\lambda\rho} - \frac{g}{2}(D_\nu F_{\alpha\beta})^b\,\partial_p^\nu f_{ab}^{\alpha\beta,\lambda\rho} \\ &\quad - gf_{abc}p^\mu A^b_\mu f_c^{\lambda\rho} - \frac{g}{2} f_{abc}F^b_{\alpha\beta} f_c^{\alpha\beta,\lambda\rho} \\ &\quad + g(F^\lambda_{b\sigma} f_{ab}^{\sigma\rho} - F^\rho_{b\sigma} f_{ab}^{\sigma\lambda}) - \frac{g}{2}(D_\nu F_{\alpha\beta})^b\,\partial_p^\nu \\ &\quad \times \int f\,\frac{[S^{\alpha\beta}, S^{\alpha\beta}]}{2}\,\frac{[Q^a, Q^b]}{2}\, dQ\, dS \end{aligned} \tag{53}$$

where

$$f_0^0(x, p) = \int f(x, p, Q, S)\, dQ\, dS, \qquad f_a^0(x, p) = \int Q^a f\, dQ\, dS$$

$$f_0^{\mu\nu}(x, p) = \int S^{\mu\nu} f\, dQ\, dS, \qquad f_a^{\mu\nu} = \int Q^a S^{\mu\nu} f\, dQ\, dS \tag{54}$$

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$$f_{ab}^{0} = \int \frac{\{Q^a, Q^b\}}{2} f\, dQ\, dS, \qquad f_{ab}^{\mu\nu} = \int \frac{\{Q^a, Q^b\}}{2} S^{\mu\nu} f\, dQ\, dS$$

Comparing Eqs. (50)–(53) with Eqs. (26), (30), (32), and (38) shows they are the same and have the following correspondence relations:

$$\begin{gathered} f_0^0 \to B_0, \quad f_a^0 \to B_a, \quad f_0^{\lambda\rho} \to T_0^{\lambda\rho}, \quad f_a^{\lambda\rho} \to T_a^{\lambda\rho} \\ f_{ab}^0 \to W_{ab}^0, \quad f_a^{\alpha\beta,\lambda\rho} \to W_a^{\alpha\beta,\lambda\rho}, \ldots \end{gathered} \tag{55}$$

Above we only give the scalar and tensor equations of the color singlet and color octet. If we define spin moments of pseudoscalar, vector, and pseudovector

$$\begin{gathered} f_{p,i}^0 = \int (-i\gamma^5 f)\, dQ\, dS, \qquad f_{\nu,i}^{\mu} = \int \gamma^\mu f\, dQ\, dS \\ f_{D,i}^{\mu} = \int \gamma^5\gamma^\mu f\, dQ\, dS \qquad (i = 0, a) \end{gathered} \tag{56}$$

we also can obtain the equations of these spin moments and the equations, are the same as those given in Section 3. Thus we see that the semiclassical model is consistent with the quantum model of the QGP.

5. SUMMARY

We have given a semiclassical model of the QGP in which the classical color charges are defined not to be commutative with each other and to satisfy the Lie algebra of the SU(3) group. The orthonormal relations of the color charge are defined. The distribution function can be expanded in the color charges in color space. The spin of the quark is described by the antisymmetric tensor $S^{\mu\nu}$, spin space is extended to 16-dimensional space, and 16 orthonormal basis vectors are introduced. The 16 basis vectors satisfy Clliford algebra and the distribution function can be expanded in spin space by these basis vectors. Using the classical transport equations, we find the transport equations of the color singlet scalar, color singlet vector, and so on, and these equations are the same as those obtained from the quantum model of the QGP. The semiclassical model remains consistent with the quantum model of the QGP.

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REFERENCES

[1] Celik, T., Engels, J., and Satz, H. (1983). *Phys. Lett. B* **129**, 323.
[2] Fucito, F., Rebbi, C., and Solomon, S. (1984). *Nucl. Phys. B* **248**, 615.
[3] Wang Enke, Li Jiarong, and Liu Lianshou. (1992). *Phys. Rev. D* **41**, 2288.
[4] Celik, T., Engels, J., and Satz, H. (1983). *Phys. Lett. B* **125**, 411.
[5] Mclerran, L. D. (1996). *Rev. Mod. Phys.* **58**, 1021.
[6] Heinz, U. (1985). *Ann. Phys.* **161**, 48.
[7] Else, H., Gyulassy, M., and Vasak, D. (1986). *Phys. Lett. B* **177**, 402.
[8] Heinz, U. (1984). *Phys. Lett. B* **144**, 228.
[9] Else, H. T., and Heinz, U. (1989). *Phys. Rep.* **183**, 81.
[10] Balescu, R. (1975). *Equilibium and Non-Equilibium Statistical Mechanics* (Wiley, New York).
[11] Lee, T. D. (1981). *Particle Physics and Introduction to Field Theory* (Harwood Academic, New York).
[12] Vasak, D., Gyulassy, M., and Else, H. (1987). *Ann. Phys.* **173**, 462.
[13] Itzylson, C., and Zuber, J. (1980). *Quantum Field Theory* (McGraw-Hill, New York).

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